MINISTRY OF EDUCATION AND TRAINING
HA NOI UNIVERSITY OF MINING AND GEOLOGY

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RESEARCH OF SOLUTIONS TO IMPROVE THE ACCURACY OF GRAVITY ANOMALIES ARE IDENTIFIED BY SATELLITE ALTIMETRY DATA ON THE GULF OF TONKIN – VIETNAM

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SUMMARY OF THE DOCTORAL THESIS

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The thesis has been completed at the **Department of Advanced Geodesy, Faculty of Geomatics and Land Administration Hanoi University of Mining and Geology**

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The thesis will be defended before the Examination Board at Hanoi University of Mining and Geology, at ....o'clock dated……………..

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INTRODUCTION

1. The necessity of the thesis

Vietnam is a country with a large sea and continental shelf with an area three times the size of the land territory. The requirements for measuring and surveying geophysical fields, including gravity fields throughout the sea, are always important and urgent tasks for marine economic development, disaster prevention, resource and environment management, security, and national defense.

For Geodesy, gravity anomalies data is used to study the Earth's shape, size, and geopotential, establishing the original data for the national coordinate system. For geophysics, gravity anomalies data is used to analyze the structure of matter in the ground, the Earth's crust, contributing to the exploration of mineral resources. For the military, gravity anomalies data is also significant in determining a ballistic missile's trajectory. For seas, gravity anomalies are closely related to seafloor topography, so it is also used to study seafloor topography.

Altimetry is one of the world's most advanced techniques. The products of Altimetry are applied in many fields such as Oceanography, Geophysics, and Geodesy ... In Oceanography, satellite Altimetry data allow us to identify ocean currents, sea whirlpools, map sea ice, and ice at the poles, monitor melting ice, and study tides. Altimetry is also used for climate research and tsunami warnings. In Geodesy, from satellite altimetry data it is possible to determine the marine Geoid, Mean Dynamic Topography (MDT), and more specifically, it is possible to identify marine gravity anomalies. Altimetry, in addition to researching the sea, is also used for inland research such as monitoring the water level of streams, reservoirs, and swamps.

Vietnam's sea gravity field can be measured by methods of direct measurement on ships and on aircraft. However, due to the complex topographical and dynamic conditions and marine meteorology, there is a high cost and long time when surveying with a dense density over a large scale. Under such conditions, determining marine gravity anomalies by indirect methods combined with applying solutions to improve the accuracy of gravity anomalies will be a feasible and highly effective solution.

Because of the above reasons, it can be seen that the choice of a doctoral thesis topic: ***"Research solutions to improve the accuracy of gravity anomalies are identified by satellite altimetry data on the Gulf of Tonkin - Vietnam"*** will have scientific value, both solve practical problems and this is also a new problem contributing to the scientific and technical development of the country.

2. Research purpose

Establish a scientific basis, and build solutions to process satellite altimetry data to improve the accuracy of determining marine gravity anomalies in general and in the sea of the Gulf of Tonkin, in particular.

Identifying marine gravity anomalies with better accuracy than ±4mGal for the Gulf of Tonkin - Vietnam by satellite altimetry data.

3. Research object

Methods of determining marine gravity anomalies by satellites altimetry data;

Computational solutions to improve the accuracy of satellite-derived gravity anomalies;

Satellite altimetry data, ship-measured gravity anomalies;

Marine gravity anomalies are determined by satellite altimetry data.

4. Research scope

On the sea of the Gulf of Tonkin - Vietnam;

For types of satellite altimetry data;

Global Geopotential Model (GGM);

Mean Dynamic Topography (MDT).

5. Research contents

Overview research of methods for determining gravity anomalies used;

Researching satellite altimetry programs, collecting and selecting satellites altimetry data in the Gulf of Tonkin - Vietnam;

The proposes some solutions to improve the accuracy of gravity anomalies identified by satellite altimetry data on the Gulf of Tonkin - Vietnam;

Programming computers for solutions to improve the accuracy of gravity anomalies is identified by satellite altimetry data;

Experimental calculations determine marine gravity anomalies by satellite altimetry data on the Gulf of Tonkin - Vietnam.

6. Research methodology

a) Statistical method: collect, synthesize and process information and documents related to research issues;

b) Analysis method: a collection of research results, analysis, and evaluation of experimental results;

c) Modeling method: research to determine the crossover of passes in satellite altimetry and research to determine the time-varying sea-surface topography;

d)) *Least Squares Collocation method*: determine marine gravity anomalies by satellite altimetry data;

e) Comparative method: assessment of the accuracy of satellite-derived gravity anomalies results;

f)) *Professional method*: receive opinions of the Supervisor, consult reputable scientists at home and abroad, and colleagues on issues in the context of research topics;

g) Informatics method: develop a program to calculate geoid height, height anomaly, and gravity anomaly from the spherical harmonic coefficients of the Global Geopotential Models.

7. The scientific and practical significance of the thesis

a) Scientific significance

Establishing a scientific basis and proposing solutions to improve the accuracy of gravity anomalies are identified by satellite altimetry data on the Gulf of Tonkin – Vietnam. Contributed to the development of the theory of determining marine gravity anomalies by satellite altimetry data.

b) Practical significance

Provide solutions to improve the accuracy of gravity anomalies are identified by satellite altimetry data on the Gulf of Tonkin - Vietnam.

The research results of the thesis will help state management agencies in researching and promulgating technical regulations on the mathematical processing of satellite altimetry data for determining marine gravity anomalies on the coast of Vietnam.

The gravity anomaly collection of the Gulf of Tonkin - Vietnam with high accuracy and resolution will make an important contribution to enriching the basis of basic survey data on this sea, valuable in the research, management, and protection of sea and island sovereignty, exploiting the potential of this sea for economic development.

8. Theoretical points to be defended

The first theoretical point: Solutions to improve the accuracy of determining marine gravity anomalies using satellite altimetry data that the thesis offers, have been theoretically rigorous and are completely feasible in empirical calculations.

The second theoretical point: Using the global geopotential model (EIGEN-6C4) mean dynamic topography (DTU15MDT), satellite altimetry data from new satellites high-precision such as Cryosat -2/GM and Saral/AltiKa/GM combined and using the Least Squares Collocation method for experimental calculations, the results of gravity anomalies are more accurate than previous studies conducted in the Gulf of Tonkin - Vietnam.

9. New points of the thesis

a) Proposing solutions and coming up with a theoretically rigorous calculation process, feasible in empirical calculations, ensure that the results of marine gravity anomalies calculated by satellite altimetry data are more accurate than previous studies carried out in the Gulf of Tonkin - Vietnam;

b) Evaluation and selection of Global Geopotential Model (EIGEN-6C4), Mean Dynamic Topography (DTU15MDT), and Satellite altimetry data from new satellites high-precision: Cryosat -2/GM and Saral/AltiKa/GM for experimental calculations, the results of marine gravity anomalies received have better accuracy than ±3mGal.

10. Structure and contents of the thesis

The structure of the thesis includes an introduction, 4 chapters, conclusions – recommendations, and 70 bibliographies. The entire content of the thesis is presented in 133 pages of A4 paper size, including 21 tables, 37 drawings, 10 charts, 3 diagrams, and 4 appendices with 19 A4 pages.

CHAPTER 1: OVERVIEW OF THE PROBLEM OF RESEARCH SOLUTIONS TO IMPROVE THE ACCURACY OF GRAVITY ANOMALIES ARE IDENTIFIED BY SATELLITE ALTIMETRY DATA

1.1. Overview of gravity survey in Vietnam

1.1.1. The concept of gravity anomalies

Gravity anomaly (Δg) at an observation point is the difference between the measured gravity value (g) and the normal gravity (γ) at that point of view and is represented by the following general formula [5], [8], [9]:

$$\Delta g = g - \gamma \qquad (1.1.1)$$

If the measured gravity values (g) and the normal gravity (γ) correspond to the same point, then the gravity anomaly is called ″homogeneous″. If the above-mentioned values correspond to different points in space, then gravity anomalies are called "mixtures". Depending on the corrections, there are types of gravity anomalies such as: Free air gravity anomaly; Faye gravity anomaly; Bughe gravity anomaly.

1.1.2. Overview of gravity survey in Vietnam [20]

1.2. Overview of the problem of research solutions to improve the accuracy of gravity anomalies are identified by satellite altimetry data

1.2.1. Overview of research works in the world

Since satellite altimetry technology begins, scientists around the world have used satellite altimetry data to build high-resolution global marine gravity field models (grid 1'x1') such as: DNSC08GRA [29]; DTU10GRAV [23]; DTU13GRAV [30]; DTU15GRAV [28]; DTU17GRAV [66].

For identifying marine gravity anomalies using satellite altimetry data, scientists around the world have used methods such as: Least Squares Collocation, Fast Fourier Technique, inverse Stokes formula, inverse Vening Meinesz formula… to identify marine gravity anomalies for local study areas and for the global.

The studies all used satellite altimetry data in geodetic mission (GM) measurement mode, combining many types of satellite altimetry to increase the density of data and improve the accuracy of determining gravity anomalies. The accuracy of the marine gravity anomaly calculated by using satellite altimetry data in some seas around the world reached about ±3.0 ÷ ±9.0 mGal, especially ±1.8 mGal.

The accuracy of marine gravity anomalies in the East Sea area of Vietnam received from the above projects is not high (approximately ±5.0mGal). Thus, the ability to apply the results of determining gravity anomalies from the world's results to Vietnam is not effective, especially in nearshore areas and shallow water areas.

Therefore, it is necessary and feasible to research solutions to improve the accuracy of gravity anomalies are identified by satellite altimetry data for experimental areas of Vietnam with higher accuracy.

1.2.2. Overview of research works in Vietnam

In Vietnam, currently, the application of satellite altimetry data to determine marine gravity anomalies for the Gulf of Tonkin - Vietnam in particular and for the East Sea has not many works. Some works only exploit the world's results, not self-study to identify marine gravity anomalies by satellite altimetry data. A small number of studies used satellite altimetry data to identify marine gravity anomalies, but for offshore areas [12]; [14]; [15]; [20].

The above Vietnamese works show that: Only one type of satellite altimetry data is used to identify marine gravity anomalies, so the point density is not enough to fully show the characteristics of the marine gravity field within the scope of research. In addition, no studies have used satellite altimetry data to identify marine gravity anomalies for the nearshore and shallow area such as the Gulf of Tonkin - Vietnam. Due to the geographical conditions in the nearshore, the shallow water area is complex, subject to many influences of waves, winds, tides, seabed topography, and other geophysical factors, so the accuracy of satellite altimetry data will be lower accuracy than offshore areas. This means that the results of identifying marine gravity anomalies by satellite altimetry data, in near-shore areas, shallow will have lower accuracy in near-shore areas and shallow water areas re, it is necessary to research solutions to improve the accuracy of determining marine gravity anomalies for nearshore and shallow waters.

1.3. Issues to continue the study of the thesis

In order to improve the accuracy of determining marine gravity anomalies by satellite altimetry data for the Gulf of Tonkin - Vietnam, the thesis will research solutions as follows:

(1) Using a combination of new types of satellites altimetry data with high accuracy, performing geodetic missions mode to increase the density of points, and increase the accuracy of determining gravity anomalies;

(2) Select the Global Geopotential Model (GGM) most suitable for the territory of Vietnam to use in the "remove - restore" technique;

(3) Select the Mean Dynamic Topography models (MDT) most suitable Vietnam for use in calculations;

(4) Select the suitable method for determining marine gravity anomalies using satellites altimetry data;

(5) Combination of satellite-derived gravity anomalies with ship-measured gravity anomalies to improve accuracy.

The above-mentioned solutions with the goal of identifying marine gravity anomalies for the Gulf of Tonkin - Vietnam by satellites altimetry data have better accuracy than ±4.0mGal and grid (1.5' x 1.5').

1.4. Conclusion of Chapter 1

In Chapter 1, the thesis presented an overview of research works in the world and Vietnam related to the problem of identifying marine gravity anomalies by satellites altimetry data and solutions to improve the accuracy of determining marine gravity anomalies, analyzed, evaluated, and pointed out the issues that the thesis needs to study.

CHAPTER 2: METHOD FOR DETERMINING MARINE GRAVITY ANOMALIES BY USING SATELLITE ALTIMETRY DATA

2.1. Satellite altimetry technique

2.1.1. Altimetry principle

The basic principle of satellite altimetry (see Figure 2.1) is based on the principle of the physics problem of calculating distances when are known velocity and time.

The distance from the satellite to the sea surface is determined by the basic formula following [54]:

$$d = c\frac{\Delta t}{2} \tag{2.1.1}$$

where, c is the velocity signal propagation; Δt is the time signal propagation 2-way; d is the distance from the satellite to the sea surface.

2.1.2. Method of determining sea surface height by satellite altimetry data

Sea Surface Height (SSH) (see Figure 2.2) is the distance from the points on the sea surface to the reference ellipsoid and is the distance difference between the satellite orbit height (H) and altimeter range (d) [54].

$$SSH = H - d \tag{2.1.2}$$

The formula for determining sea surface height after correction (SSH_{corr}):

$$SSH_{corr} = H - (d + e) \tag{2.1.3}$$

where (e) is corrections for the measured distance from the satellite to the sea surface.

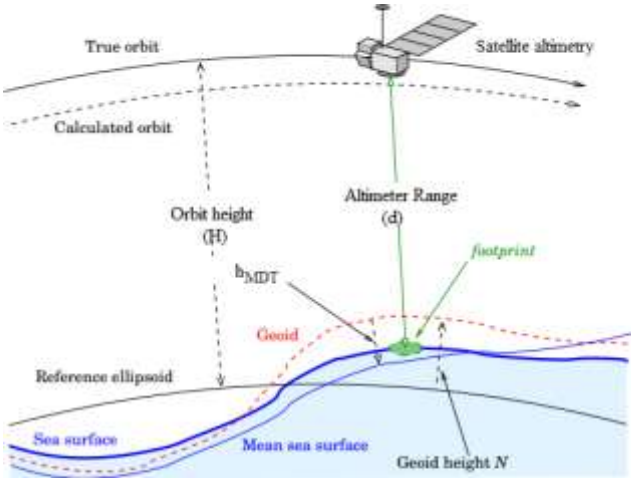


Figure 2.1. The principle of satellite altimetry (source [62])

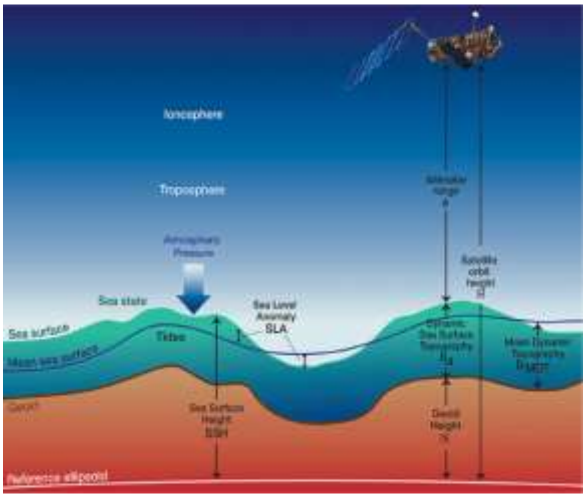


Figure 2.2. The relationship between sea surface height and satellite measurements (source: Internet)

2.1.3. The corrections of satellite altimetry

According to the documents [4], [20], [51] the corrections (e) in expression (2.1.3) include the following groups of correction (see Figure 2.4):

- Instrumental corrections;
- Propagation corrections;
- Surface corrections;
- Geophysical corrections.

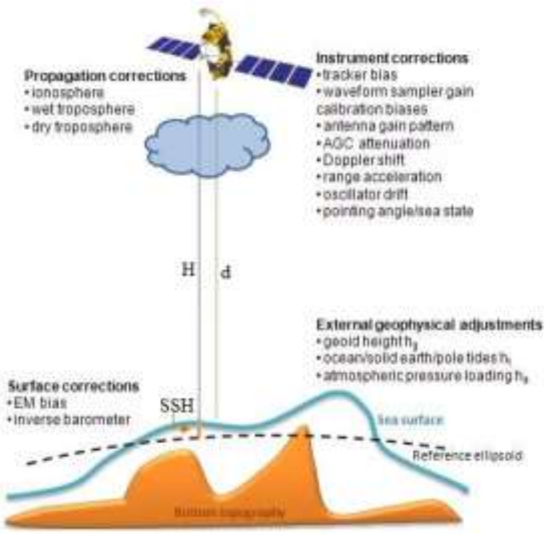


Figure 2.4. Altimetry corrections illustration (source [51])

The formula for calculation sea surface height [54]:

$$SSH_{corr} = H - (d + e_{Ins} + e_{Ion} + e_{Wet} + e_{Dry} + e_{EMBias} + e_{IB} + e_{O\_tide} + e_{P\_tide} + e_{E\_tide}) \tag{2.1.16}$$

where: H, d, e_{Ins}, e_{Ion}, e_{Wet}, e_{Dry}, e_{EMBias}, e_{IB}, e_{O_tide}, e_{P_tide}, e_{E_tide} respectively, the satellite orbit height, the altimeter range, the instrumental correction, the ionosphere correction, the tropospheric (wet, dry) corrections, the electromagnetic bias correction, the inverse barometer correction, the tides (ocean, pole, earth) corrections.

2.1.4. Altimetry missions and altimeter data

2.1.4.1. Altimetry missions

- Past missions: Skylab, GEOS 3, Seasat, Geosat, ERS-1, TOPEX/ Poseidon, ERS-2, GFO, Jason-1, Envisat, Jason-2, HY-2A, Spot-1÷ Spot-5.
- Current missions: Cryosat-2/GM, Saral-DP/ AltiKa, Sentinel-3A, Jason-3, Sentinel-3B, CFOSAT, HY-2B, HY-2C, HY-2D, Jason-CS / Sentinel-6MF.
- Future missions: Swot

2.1.4.2. Altimeter data

Three levels of processing of altimeter data [33], [54]:

• Level-0 (L0) is the raw telemetered data;

• Level-1 (L1) is the Level-0 data corrected for instrumental effects;

• Level-2 (L2) is the Level-1 data corrected for geophysical effects.

2.2. Method of determining marine gravity anomalies by using satellite altimetry data

2.2.1. Methodology

Formula for computing the geoid height using satellite altimetry data as follows:

$$N = SSH - (h_{MDT} + h_t) \tag{2.2.3}$$

According to the documents [23], [28], [62] the gravity anomaly (Δg), geoid height (N) related to the disturbing potential (T) by these formulas:

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r}T \tag{2.2.4}$$

$$N = \frac{T}{\gamma} \tag{2.2.5}$$

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r}T \approx -\frac{1}{\gamma}\left(\frac{\partial N}{\partial r} + \frac{2}{r}N\right) \tag{2.2.6}$$

Thus, from the geoid height (N) in the formula (2.2.3) we will calculate the gravity anomaly using the formula (2.2.6).

The geoid height (N) can be described in terms of a long wavelength reference geoid (N_{EGM}), and residuals (ΔN).

Then, from the formula (2.2.3) we have the formula for determining the residual geoid height (ΔN) as follows:

$$\Delta N = SSH - (N_{EGM} + h_{MDT} + h_t) \tag{2.2.7}$$

The residual geoid height (ΔN) will be used to determine the residual gravity anomaly (δg) by methods such as: Least Squares Collocation, Fast Fourier Technique, inverse Stokes formula, inverse Vening Meinesz formula. Then, restoring the long-wavelength gravity anomaly (Δg_{EGM}) using the fully normalized gravitational coefficients of global geopotential model according to the "remove-restore" technique. The final result will get a gravity anomaly (Δg) using the formula:

$$\Delta g = \delta g + \Delta g_{EGM} \tag{2.2.8}$$

2.2.2. The process of determining marine gravity anomalies by satellite altimetry data

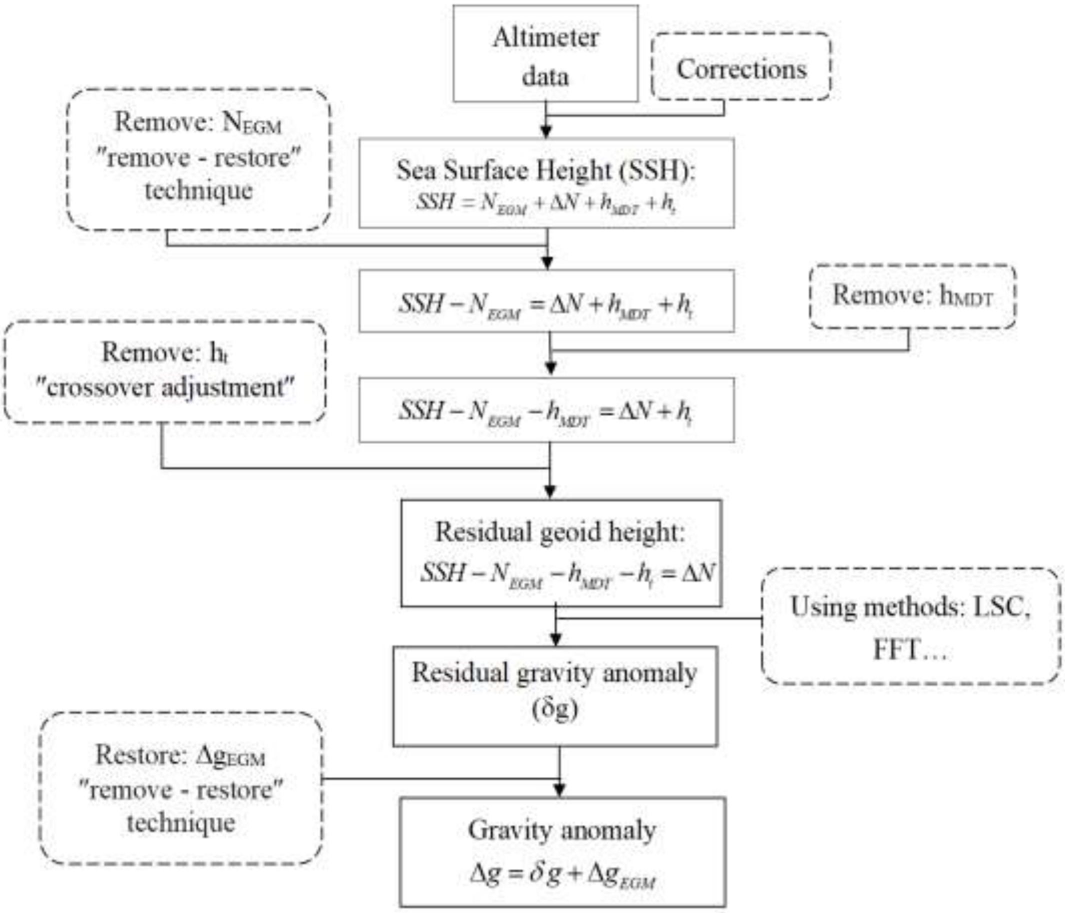


Diagram 1. The process of determining marine gravity anomalies by satellite altimetry data

2.2.3. Method for determining geoid height and gravity anomalies from the normalized gravitational coefficients of global geopotential model

The general formula for determining geoid height from the fully normalized gravitational coefficients of global geopotential model [43], [46], [50], [59]:

$$N = \zeta_0 + \frac{GM}{r.\gamma}\left[\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\bar{C}_{nm}\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)\right)\bar{P}_{n,m}(\sin\bar{\varphi})\right] + \frac{\Delta g_B}{\bar{\gamma}} h \qquad (2.2.16)$$

Note: $\zeta_0 = \frac{GM - GM_0}{r.\gamma} - \frac{W_0 - U_0}{\gamma}$ is the zero degree term

The general formula for determining gravity anomalies from the fully normalized gravitational coefficients of global geopotential model:

$$\Delta g = \Delta g_0 + \frac{GM}{r^2}\left[\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n (n-1)\sum_{m=0}^{n}\left(\bar{C}_{nm}\cos(m\lambda) + \bar{S}_{nm}\sin(m\lambda)\right)\bar{P}_{n,m}(\sin\bar{\varphi})\right] \qquad (2.2.18)$$

Note: $\Delta g_0 = -\frac{GM - GM_0}{r^2} + \frac{2(W_0 - U_0)}{r}$

where, GM: Earth's gravitational constant; GM_0: mass gravity constant of the reference ellipsoid; r: distance from the point to the mass Earth's center; γ : normal gravity on the ellipsoidal surface; a: semi-major axis of reference ellip- soid; $\bar{\varphi},\lambda$: geocentric latitude and geocentric longitude (geodetic longitude), respectively; $\bar{C}_{nm}$ và $\bar{S}_{nm}$ fully normalized gravitational coefficients; n, m: degree and order of spherical harmonic, respectively; $\bar{P}_{n,m}(\sin\bar{\varphi})$: fully normalized associated Legendre functions; Δg_0: zero degree term.

Before calculating, the data types need to agree on the same tide system. The formula for converting geoid height and gravity anomaly between tide systems as follows [37], [50]:

a) The relationship of geoid height between tide systems:

$$N_m = N_z + 9.9 - 29.6\sin^2\bar{\varphi}(cm) \quad (2.2.37)$$

$$N_z = N_n + k(9.9 - 29.6\sin^2\bar{\varphi})(cm) \quad (2.2.38)$$

$$N_n = N_m + (1+k)(-9.9 + 29.6\sin^2\bar{\varphi})(cm) \quad (2.2.39)$$

b) The relationship of gravity anomaly between tide systems:

$$\Delta g_m = \Delta g_z - 30.4 + 91.2\sin^2\bar{\varphi}(\mu Gal) \quad (2.2.40)$$

$$\Delta g_z = \Delta g_n + (\delta - 1)(-30.4 + 91.2\sin^2\bar{\varphi})(\mu Gal) \quad (2.2.41)$$

$$\Delta g_n = \Delta g_m - \delta(-30.4 + 91.2\sin^2\bar{\varphi})(\mu Gal) \quad (2.2.42)$$

where, k and δ: love coefficients (k=0.3; δ=1.16); $N_n, \Delta g_n$: tide-free geoid height and gravity anomaly, respectively; $N_m, \Delta g_m$: mean tide geoid height and gravity anomaly, respectively; $N_z, \Delta g_z$: zero-tide geoid height and gravity anomaly, respectively; $\bar{\varphi}$: geocentric latitude.

2.2.4. The mean dynamic topography

The mean dynamic topography (h_{MDT}) is the difference between the mean sea surface (h_{MSS}) and the geoid heights (N) are shown in Fig. 2.2. It is calculated from the following formula:

$$h_{MDT} = h_{MSS} - N \quad (2.2.43)$$

2.2.5. Removing the time varying sea surface topography using the crossover adjustment method

The dynamic sea surface topography can be described in terms of a mean dynamic topography (h_{MDT}) and a time varying sea surface topography (h_t). In order to remove the time varying sea surface topography can use to the crossover adjustment method [58], [62]. The equations in the crossover adjustment as follows:

(1) For areas with short tracks (L<1000km):

$$\begin{cases} v_{ij} = a_j - a_i + dH_{ij} \\ V_{ki} = -a_k + SSH'_{ki} \end{cases} \quad (2.2.56)$$

(2) For areas with medium tracks (L<2000km):

$$\begin{cases} v_{ij} = (a_j + b_j.\mu_i) - (a_i + b_i.\mu_j) + dH_{ij} \\ V_{ki} = -(a_k + b_k.\mu_{ki}) + SSH'_{ki} \end{cases} \quad (2.2.57)$$

(3) For areas with long tracks (L>2000km):

$$\begin{cases} v_{ij} = (a_j + c_j.\sin\mu_i + d_j.\cos\mu_i) - (a_i + c_i.\sin\mu_j + d_i.\cos\mu_j) + dH_{ij} \\ V_{ki} = -(a_k + c_k.\sin\mu_{ki} + d_k.\cos\mu_{ki}) + SSH'_{ki} \end{cases} \quad (2.2.58)$$

The parameters a, b, c, d is solved according to the principle of the smallest number of squares with condition:

$$\sum v^T P v + \mathrm{w}\sum V_k^T P_k V_k = \min \quad (2.2.59)$$

where, *w* is the relative weight.

2.3. Conclusion of Chapter 2

In chapter 2, the altimetry principle was presented, the corrections of satellite altimetry, altimetry missions, and altimeter data. In addition, the thesis detailed the theoretical basis of the method and process of determining marine gravity anomalies by using satellite altimetry data. The theory of the method will be the basis for proposing appropriate solutions to improve the accuracy of determining gravity anomalies by satellite altimetry data.

CHAPTER 3: SOLUTIONS TO IMPROVE THE ACCURACY OF DETERMINING MARINE GRAVITY ANOMALIES BY SATELLITE ALTIMETRY DATA

3.1. Theoretical basis for solutions to improve the accuracy of determining marine gravity anomalies by satellite altimetry data

3.1.1. Theoretical basis

The method for determining marine gravity anomalies using satellite altimetry data is detailed in section 2.2. The residual geoid height (ΔN) determined by formula (2.2.7) will be used to determine the residual gravity anomaly (δg) by methods such as: Least Squares Collocation, Fast Fourier Technique, inverse Stokes formula, inverse Vening Meinesz formula…Thus, the accuracy of the residual gravity anomaly will depend on the accuracy of the components: SSH, N_{EGM}, h_{MDT}, h_t.

Therefore, in order to improve the accuracy of the results of marine gravity anomalies determined by satellite altimetry data, the thesis will propose the solutions following:

(1) Select data of satellites altimetry performing geodetic missions and combination to receive high-precision Sea Surface Height (SSH) data as well as increase the density of points for the research area;

(2) Using GPS/leveling data to evaluate and select the Global Geopotential Model (GGM) most suitable for the territory of Vietnam to receive the high-precision long-wavelength geoid height (N_{EGM}) for use in the "remove - restore" technique;

(3) Using the tidal gauge data along the coast of Vietnam with many years of monitoring time to evaluate and select the Mean Dynamic Topography models (MDT) most suitable Vietnam for use in calculations;

(4) Selection the best method for converting the residual geoid heights into residual gravity anomalies;

(5) Using the ship-measured gravity anomalies to evaluate the results as well as to fit with satellite-derived gravity anomalies.

The solutions will be presented more clearly in the next sections.

3.1.2. Process to improve the accuracy of identifying marine gravity anomalies using satellite altimetry data

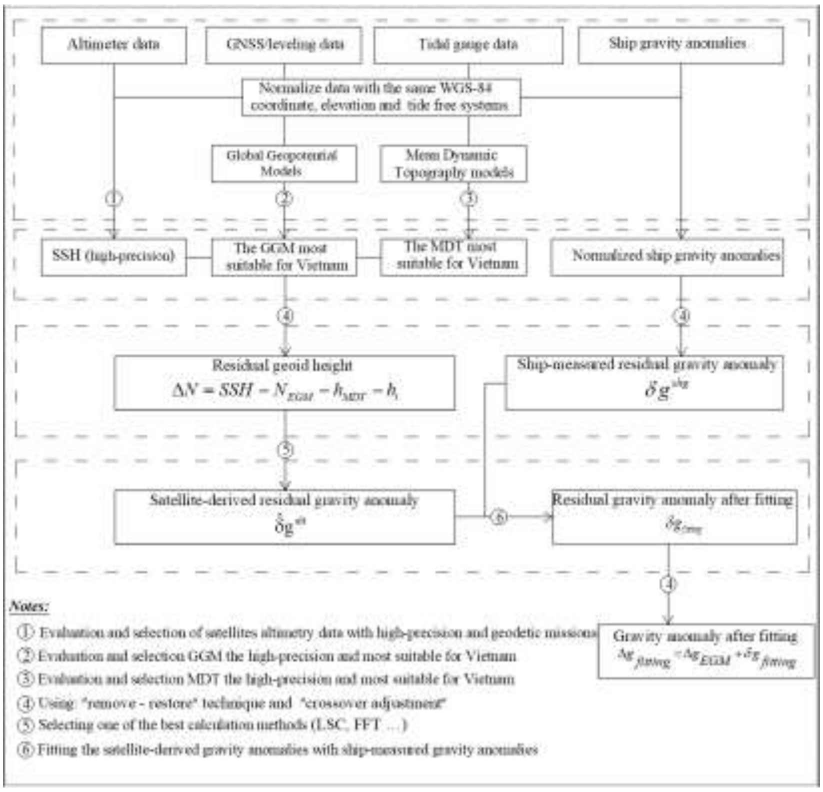


Diagramm 2. Process to improve the accuracy of identifying marine gravity anomalies using satellite altimetry data

3.2. Solution 1 - combines high-precision satellites altimetry data, performing geodetic missions to improve the accuracy of marine gravity anomalies determination

3.2.1. Selection of satellites altimetry data, performing geodetic missions

a) Exact Repeat Mission

Exact Repeat Mission is understood as a mission where the position of data at different measurement cycles will not change and be repeated relatively accurately. The orbital deviation between 2 consecutive cycles is within ± 1km [33].

Therefore, using the data of satellites when performing Exact Repeat Mission to study in detail the variation of the marine gravity field will not be feasible in terms of measurement point density.

b) Geodetic Mission

Geodetic Mission is understood as a mission where the position of data at different measurement cycles will always change. Because, the position of the data at different measurement cycles is always changing and tends to intertwine in the direction from West to East creating a dense network of measurement points. Therefore, after many observation cycles of a satellite, the distance between the measurements at the equator reaches from 4km ÷ 8km corresponding to the point density of (3.5'x3.5'). Thus, if the data of satellites with Geodetic Mission is selected to study in detail the variation of the marine gravity field, the point density has been greatly increased compared to the Exact Repeat Mission.

3.2.2. Combines high-precision satellites altimetry data together

The satellites altimetry that has been performing Geodetic Mission includes 6 satellites: Geosat, ERS-1, J1, HY-2A, Cryosat-2/GM và SARAL-DP/AltiKa. Data of these types of satellites meet the requirements of sea surface height point density. The Root mean square (RMS) about SSH of the 6 types of satellites performing Geodetic Missions is given in Table 3.3.

Table 3.3. Statistics of errors determining sea surface height of satellites performing Geodetic Missions [35], [36], [69].

Error	Geosat	ERS-1	Jason-1	Cryosat-2/GM	HY-2A	SARAL/ AltiKa
Altimeter noise (cm)	5.0	3.0	1.6	0.2	-	1.0
Ionosphere (cm)	2.0-3.0	2.0-3.0	0.5	0.2	-	0.3
Sea state bias (cm)	2.0	2.0	2.0	0.2	-	2.0
Dry troposphere (cm)	1.0	1.0	0.7	0.2	-	0.7
Wet Troposphere(cm)	4.0	1.2	1.2	0.2	-	1.0
Altimeter range after corrections (cm)	7.0	4.6	3.0	0.4	4.0	2.6
Orbit (cm)	20.0	18.0	1.5	1.0	3.0	2.0
Sea surface height (cm)	**21.0**	**18.6**	**3.3**	**1.0**	**5.0**	**3.2**

Based on Table 3.3, SARAL/AltiKa and Cryosat-2/GM satellites are two satellites with higher sea surface height accuracy than other satellites. In addition, the SARAL/AltiKa and Cryosat-2/GM satellites have different orbital tilt angles. Therefore, if you combine many data cycles of these 2 types of satellites together, it will completely meet the requirements of measurement point density as well as the accuracy requirements when using satellite altimetry data to study the characteristics of the marine gravity field.

3.3. Solution 2 - select the Global Geopotential Model most suitable for the territory of Vietnam

3.3.1. Results of evaluation of the accuracy of the Global Geopotential Models

According to the results of the International Centre for Global Earth Models review, the models: EGM2008 [52]; GECO [42]; EIGEN-6C4 [40]; SGG-UGM-1 [63], [64] are 4 of the models with degree (n) and order (m) up to 2190 and high accuracy. The root mean square of the above-mentioned models is: ±0.1877m, ±0.1763m; ±0.1780m; ±0.1764m. Therefore, use these models to evaluate and find the model that suits Vietnam.

3.3.2. Establish a program for calculating geoid height, height anomalies, and gravity anomalies from the fully normalized gravitational coefficients of global geopotential models

On the basis of the formulas (2.2.16) and (2.2.18), establish a program for calculating geoid height, height anomalies, and gravity anomalies from the fully normalized gravitational coefficients of global geopotential models for any points is known when coordinate components (B, L, H). The program is called **Geomat2015** [18], [70]. The block diagram of the program is shown in Figure 3.5.

The **Geomat201**5 program is a combination of 9 subroutines written in the Matlab programming language.

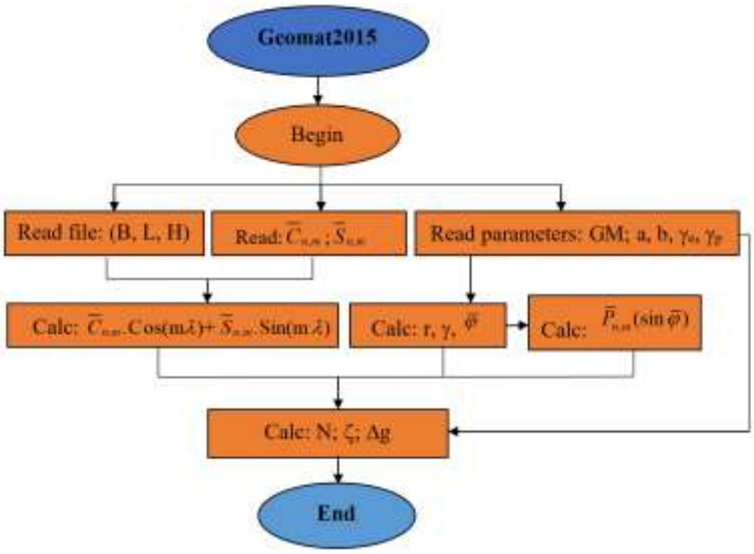


Figure 3.5. Diagram of the program **Geomat2015**

Figure 3.6. The GPS/levelling and tidal gauge station's locations in the territory of Vietnam

3.3.3. Evaluation of the accuracy of the Global Geopotential Models EGM2008, GECO, EIGEN-6C4, and SGG-UGM-1 in the territory of Vietnam

For the territory of Vietnam, use GPS/levelling data to assess the accuracy of these 4 models. The GPS/leveling data source consists of 818 points, evenly distributed over the territory of Vietnam (see Figure 3.6). At each GPS/leveling point, there are geodetic coordinates and elevations (B, L, H_{GPS}) in the WGS-84 coordinate system, as well as elevation $h_{levelling}$ in the national elevation system with the sea surface height at Hon Dau tidal gauge station (Hai Phong) [10].

Before evaluating the accuracy, filter out the raw values in the range of measurements with the following criteria: filter out the > (mean + 3*standard deviation) and < (mean - 3*standard deviation). The results of the accuracy assessment are set out in Table 3.5.

Table 3.5. Summary of the results comparing height anomalies calculated from the spherical harmonic function coefficients of the Global Geopotential Models EGM2008, GECO, EIGEN-6C4, SGG-UGM-1 with height anomalies calculated from GPS/levelling data.

No.	Statistics	EGM2008	GECO	EIGEN-6C4	SGG-UGM-1
		ζ (m)	**ζ (m)**	**ζ (m)**	**ζ (m)**
		Tide free			
1	Min. Dev: $\delta_\zeta^{\min}$	-1.650	-1.401	-1.433	-1.354
2	Max. Dev: $\delta_\zeta^{\max}$	+0.036	-0.328	-0.0310	-0.252
3	Mean. Dev: δ_ζ^{mean}	-0.802	-0.873	-0.886	-0.845

No.	Statistics	EGM2008	GECO	EIGEN-6C4	SGG-UGM-1
		ζ (m)	ζ (m)	ζ (m)	ζ (m)
		Tide free			
4	Std. Dev: σ_ζ	±0.280	±0.183	±0.179	±0.180

Based on the results of the accuracy assessment of 04 Global Geopotential Models mentioned in Table 3.5, the EIGEN-6C4 is the model with high accuracy and is most suitable for the territory of Vietnam.

3.4. Solution 3- select the Mean Dynamic Topography model suitable for Vietnam

The evaluation of the accuracy of the Mean Dynamic Topography model in Vietnam is based on the data from 31 tidal gauge stations (within the coordinates: $8.5^0 < B < 21.5^0$N; $103.5^0 < L < 109.5^0$E). Số liệu các trạm nghiệm triều được tham khảo trong tài liệu [6]. Data for tidal gauge stations are referenced in the [6]. Location of tidal gauge stations (see Figure 3.6).

Table 3.6. Statistics table of results of the assessment of the accuracy of MDT models in the territory of Vietnam

No.	Statistics	DNSC08 MDT	DTU10MDT	DTU13MDT	DTU15MDT
		Tide free			
1	Min. Dev: $\delta_{h_{MDT}}^{\min}$ (m)	+0.350	+0.641	+0.869	+0.794
2	Max. Dev: $\delta_{h_{MDT}}^{\max}$ (m)	+1.182	+1.366	+1.444	+1.361
3	Mean. Dev: $\delta_{h_{MDT}}^{tb}$ (m)	+0.900	+1.100	+1.123	+1.053
4	Std. Dev: $\sigma_{h_{MDT}}$ (m)	±0.208	±0.172	±0.132	±0.131

Based on the results of the accuracy assessment of MDT models, the DTU15MDT model is the most accurate model and suitable for Vietnam's waters.

3.5. Solution 4 - Selection of a method for converting the residual geoid heights into residual gravity anomalies

The Earth's gravity field is a diverse statistical object, so it is impossible to obtain regions with the same gravitational characteristics. After removing the long wavelength as well as the influence of the local terrain (convex, concave compared to the local surface), the remaining data field is called a "residual field". There are many methods for converting residual geoid heights into residual gravity anomalies, such as the Least Squares Collocation method, FFT, the reverse Stokes formula and the reverse Vening–Meinesz formula…

Among the methods mentioned above, the Least Squares Collocation method has more superiority in solving the problems of physical geodesy, this method allows the simultaneous use of combinations of different types of data, while the reverse Stokes formula and the reverse Vening-Meinesz formula use only one type of metric. In integral formulas, the processing is performed with the input data in the form of values, therefore interpolation calculations often have to be used; whereas in the Least Squares Collocation method linear transformations are carried out in the form of covariance functions and on the basis of strict analysis. Not only that, the mathematical transformations in the Least Squares Collocation method are essentially differential operations, so they are simpler than integrals.

Therefore, the thesis will choose the Least Squares Collocation method to convert the residual geoid heights into the residual gravity anomalies.

3.5.1. Theoretical basis of the Least Squares Collocation method

The theoretical basis of the Least Squares Collocation method is as follows: Assume that there are two sets of random quantities [3], [5]:

The set "measured" $L_1, L_2, ...L_q$, denoted by the q-dimensional vector:

$$L = \left[L_1 L_2 ... L_q\right]^T \tag{3.5.1}$$

and the set of "signals" to be defined as $S_1, S_2, ...S_m$, is denoted by the m-dimensional vector:

$$S = \left[S_1 S_2 ... S_m\right]^T \tag{3.5.2}$$

thus the vectors L and S are both column vectors

The best linear estimation of the vector S is denoted as $\widehat{S}$, written in the form of:

$$\widehat{S} = C_{SL} C_{LL}^{-1} L \tag{3.5.20}$$

This is the best linear estimate of the "signal" vector as a linear function of the observed metric vector L (the non-deviant estimate with the smallest variance).

The formula (3.5.20) is called the Least Squares Collocation formula.

3.5.2. Apply the Least Squares Collocation method to determine the residue marine gravity anomaly using satellite altimetry data

Suppose there are n residual geoid height values $\Delta N_1, \Delta N_2, ..., \Delta N_n$ defined by the formula (2.2.7). According to [15], [20], [21], [22], [34], [57], [58], [62] and applying the formula (3.5.20) we have a residual gravity anomalies value at point P (δg_P) calculated using the formula:

$$\delta g_P = C^T_{\Delta N \delta g_P} . \left[C_{\Delta N \Delta N} + C_\Delta\right]^{-1} . \Delta N \tag{3.5.21}$$

The accuracy of the received residual gravity anomalies is evaluated by the formula:

$$\sigma^2_{\delta g_P} = C_{\delta g_P \delta g_P} - C^T_{\Delta N \delta g_P} . \left[C_{\Delta N \Delta N} + C_\Delta\right]^{-1} . C_{\Delta N \delta g_P} \tag{3.5.22}$$

where:

$C_{\Delta N \delta g_P}$ is the cross-covariance matrix between the residual gravity anomalies to be determined and the residual geoid heights;

$C_{\Delta N \Delta N}$ is the covariance matrix of the residual geoid heights;

$C_\Delta = \sigma^2 . E$ is the measurement variance matrix (σ^2 which is the variance of the sea surface height (SSH), E is the unit matrix (class n));

$C_{\delta g_P \delta g_P}$ is the covariance matrix of the residual gravity anomalies.

3.1. Solution 5 – fitting the satellite-derived gravity anomalies with ship-measured gravity anomalies

Due to various reasons, the results of calculating marine gravity anomalies by satellite altimetry data have a difference of one quantity (ε) from ship - measured gravity anomalies. This difference includes: system difference (ε_{ht}) and random difference (ε_{nn}) [57].

$$\varepsilon = \varepsilon_{ht} + \varepsilon_{nn} \tag{3.6.1}$$

The system difference (ε_{ht}) is determined by the following formula:

$$\varepsilon_{ht} = \frac{\sum_{i=1}^{m} (\delta_i^{shg} - \delta_i^{shg-alt})}{m} \tag{3.6.2}$$

where, δ_i^{shg} and $\delta_i^{shg-alt}$ are the parallel values between the ship-measured gravity anomalies and the satellite-derived gravity anomalies, m is the total number of parallel points.

Due to the nature of the Least Squares Collocation method, before fitting between satellite-derived gravity anomalies with ship-measured gravity anomalies by the Least Squares Collocation method, we need to fit the first system deviation value ε_{ht} to the satellite-derived gravity anomalies using the formula:

$$\delta_i^{alt} = \varepsilon_{ht} + \delta_i^{alt}, \text{ with } i=1,2,3\ldots,k. \tag{3.6.3}$$

Fitting second time: Using the Least Squares Collocation method to fit the random deviation ε_{nn} to the satellite-derived gravity anomalies.

According to the formula (3.5.20) we have the residual gravity anomalies of point P (δg_P) after fitting calculated using the formula [15, 20, 34, 58, 62]:

$$\delta g_P = \begin{bmatrix} C_{\delta g^{alt} \delta g_P} \\ C_{\delta g^{shg} \delta g_P} \end{bmatrix}^T \begin{bmatrix} C_{\delta g^{alt} \delta g^{alt}} + C_{\Delta^{alt}} & C_{\delta g^{alt} \delta g^{shg}} \\ C^T_{\delta g^{alt} \delta g^{shg}} & C_{\delta g^{shg} \delta g^{shg}} + C_{\Delta^{shg}} \end{bmatrix}^{-1} \begin{bmatrix} \delta g^{alt} \\ \delta g^{shg} \end{bmatrix} \tag{3.6.4}$$

The following is a fitting procedure for satellite-derived gravity anomalies with ship-measured gravity anomalies.

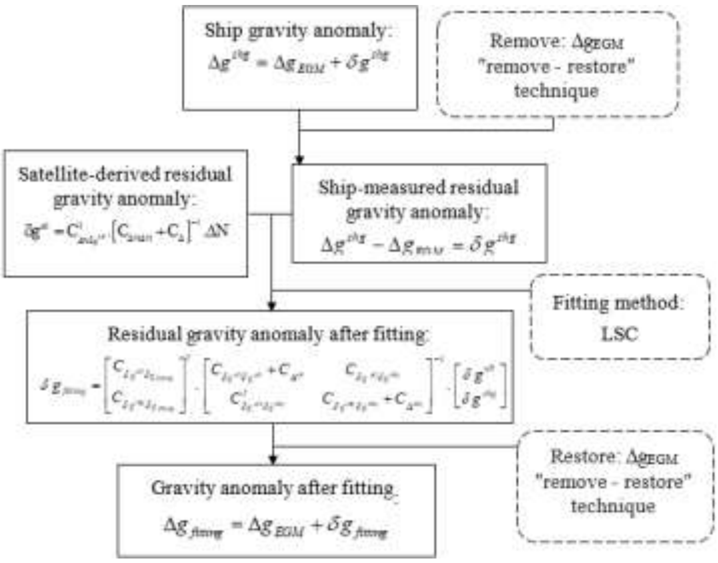


Diagram 3. Diagram of the method of fitting the satellite-derived gravity anomalies with ship-measured gravity anomalies

3.6. Conclusion of Chapter 3

- Combining cryosat-2/GM and SARAL/AltiKa/GM satellite data together will produce high-precision Sea Surface Height (SSH) data as well as increase the density of points for the research area;

- The Global Geopotential Model (EIGEN-6C4) is the most suitable for the territory of Vietnam for use in the "remove-restore" technique;

- The Mean Dynamic Topography model (DTU15MDT) is most suitable Vietnam for use in calculations;

- The Least Squares Collocation method is the best for converting the residual geoid heights into residual gravity anomalies

- Combining the satellite-derived gravity anomalies with ship-measured gravity anomalies eliminates system error and improves the accuracy of satellite-derived gravity anomalies.

CHAPTER 4: APPLYING SOLUTIONS TO IMPROVE THE ACCURACY OF IDENTIFYING MARINE GRAVITY ANOMALIES BY SATELLITE ALTIMETRY DATA ON THE GULF OF TONKIN - VIETNAM

4.1. Research areas and empirical data

4.1.1. Research areas

The experimental research area of the thesis is the Gulf of Tonkin - Vietnam (see Figure 4.2). The scope of empirical calculations of the thesis is limited by geographical coordinates: $16^{0}30' \leq B \leq 22^{0}00'$ and $105^{0}30' \leq L \leq 108^{0}30'$.

4.1.2. Empirical data

4.1.2.1. Satellite altimetry data

Satellite altimetry data provided by Archiving, Validation and Interpretation of Satellite Oceanographicthe (AVISO) include: 105 cycles (from cycle 5 – July 31, 2010, to cycle 109 – September 22, 2018) corresponding to 30810 measurement points of the Cryosat-2/GM satellite and 54 cycles (from cycle 36 – July 4, 2016, to cycle 89 – September 6, 2021) corresponding to 16062 measurement points of the Saral/AltiKa satellite. The total number of sea surface height points measured by Cryosat - 2/GM and Saral/AltiKa satellites in the Gulf of Tonkin -Vietnam is 46872 points. At each of the above-mentioned measurement points, geodetic coordinates (B, L) of the WGS-84 coordinate system and sea surface height (SSH) have been corrected for errors and belong to the free tide. Given the number of measurement points and the distribution of passes of the two types of satellites mentioned above, the satellite altimetry data available in the study area has a corresponding density (1.5'x1.5') (see Figure 4.2).

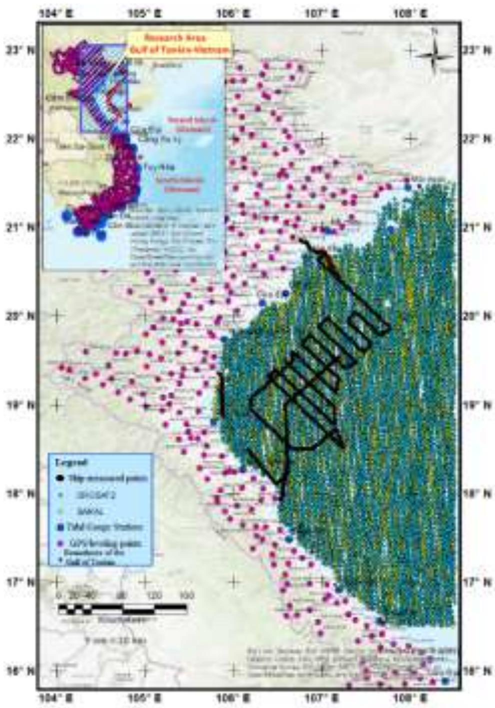


Figure 4.2. Location of empirical data

4.1.2.2. Ship-measured gravity anomalies data

Table 4.2. Table summarizing the types of data used in the thesis.

Data source	Number of points	Mean	Min, Max	Coordinate system	Tide System
SSH – Cryosat-2/GM (m)	30810	-18.609	-23.882 -10.336	WGS-84	Tide free
SSH - Saral/Altika (m)	16062	-18.643	-23.768 -10.056		
Ship-measured gravity anomalies data for evaluation: $\Delta g_{KKTD} = g - \gamma_0$ (mGal)	56978	-31.014	-61.460 +27.912		
Ship-measured gravity anomalies data for fitting: $\Delta g_{KKTD} = g - \gamma_0$ (mGal)	2011	-30.219	-59.176 +11.210		

To demonstrate the solutions to improve the accuracy proposed in chapter 3 with a strictly theoretical basis and ensure to improve the accuracy of determining marine gravity anomalies by satellite altimetry data on the Gulf of Tonkin - Vietnam. In chapter 4, empirical calculations will be carried out with options as follows:

- ***Option 1****:* Using 105 cycles of the Cryosat-2/GM satellite; Global Geopotential Model (EIGEN-6C4) and Mean Dynamic Topography model (DTU15MDT) to determine marine gravity anomalies by the Least Squares Collocation method.
- ***Option 2***: Using 54 cycles of the Saral/AltiKa satellite; Global Geopotential Model (EIGEN-6C4) and Mean Dynamic Topography model (DTU15MDT) to determine marine gravity anomalies by the Least Squares Collocation method.
- ***Option 3****:* Combine 105 cycles of the Cryosat-2/GM satellite with the 54 cycles of the Saral/AltiKa satellite; Using Global Geopotential Model (EIGEN-6C4) and Mean Dynamic Topography model (DTU15MDT) to determine marine gravity anomalies by the Least Squares Collocation method.

4.2. Empirical results of options.

4.2.1. Experimental determination of residual geoid height.

Applying the calculation procedure outlined in Diagram 1 and Diagram 2, we have the residual geoid height of the options summarized in Table 4.3.

Table 4.3. Table of results for calculating the residual geoid height of the options

Options	Satellites	Number of points	Tide System	$\Delta N = SSH - N_{EIGEN-6C4} - h_{MDT15} - h_t$ (residual geoid height)		
				Mean (m)	Min (m)	Max (m)
Option 1	Cryosat-2/GM	30810	Tide free	+0.112	-0.496	+0.705
Option 2	Saral/AltiKa	16062		+0.134	-0.656	+0.673
Option 3	Cryosat-2/GM & Saral/AltiKa	46872		+0.120	-0.656	+0.705

4.2.2. Determination of residual marine gravity anomaly by satellite altimetry data

4.2.2.1. Determination of analytic covariance function parameters from residual geoid height

The results of determining the parameters of the analytic covariance function of options are given in Table 4.5

Table 4.5. The results of determining the parameters of the analytic covariance function of options

Options	N	a	R_B-R (km)	A (m/s)^4	Gravity anomaly variance (mGal^2)
Option 1	320	25.4263	-1.000	$0.1451.10^{-2}$	60.88
Option 2	315	26.8046	-1.000	$0.1217.10^{-2}$	58.96
Option 3	320	25.2415	-1.000	$0.1451.10^{-2}$	60.44

4.2.2.2. Determination of residual marine gravity anomalies from residual geoid height.

The results of marine gravity anomalies received in the Gulf of Tonkin - Vietnam according to the options statistically in Table 4.6.

Table 4.6. Statistics summarizing the results of residual marine gravity anomalies (δg) determined by satellite altimetry according to the options

Options	Satellites	Grid	Tide System	Residual marine gravity anomalies (δg)		
				Mean (mGal)	Min (mGal)	Max (mGal)
Option 1	Cryosat-2/GM	2′x2′	Tide free	-0.85	-20.37	+20.88
Option 2	Saral/AltiKa	2′x2′		-1.05	-19.88	+18.02
Option 3	Cryosat-2/GM & Saral/AltiKa	1.5′x1.5′		-0.86	-20.17	+20.79

4.2.2.3. Assess the accuracy of the results of determining the residual marine gravity anomalies from residual geoid height on the Gulf of Tonkin - Vietnam.

Using 56978 points marine gravity anomalies measured directly by ships on the Gulf of Tonkin - Vietnam to assess the accuracy of the results of determining the residual marine gravity anomalies by satellite altimetry data of 3 options.

After filtering out the raw measurements in the range of measurements, an accuracy assessment is carried out. The results of the accuracy assessment are shown in Table 4.7.

Table 4.7. Summary of the results comparing the residual satellite-derived gravity anomalies with the residual ship-measured gravity anomalies of 3 options on the Gulf of Tonkin - Vietnam

No.	Statistics	Option 1	Option 2	Option 3
1	Min. Dev: $\Delta_{\delta g}^{\min}$ (mGal)	-7.43	-7.60	-7.32
2	Max. Dev: $\Delta_{\delta g}^{\max}$ (mGal)	+11.63	+12.04	+11.59
3	Mean. Dev: $\Delta_{\delta g}^{tb}$ (mGal)	+2.12	+2.27	+2.16
4	Std. Dev: $\sigma_{\delta g}$ (mGal)	±2.80	±2.89	±2.77

The comparative results show that the accuracy of the residual satellite-derived gravity anomalies according to option 3 has a higher accuracy and resolution than option 1 and option 2 (assessed according to the achieved standard deviation: ±2.77 mGal).

4.2.3. Fitting satellite-derived gravity anomalies with the ship- measured gravity anomalies using the Least Squares Collocation method.

4.2.3.1. Fitting the first time (system deviation correction)

The mean deviation between the residual satellite-derived gravity anomalies and residual ship-measured gravity anomalies according to the options is: +2.12 mGal (Option 1); +2.27 mGal (Option 2); +2.16 mGal (Option 3). This is the system deviation value that we need to look for.

Applying the formula (3.6.3) we have the values of residual satellite-derived gravity anomalies after the fitting the first time of the three options as follows:

Option 1: $\delta_i^{alt} = \widehat{\delta}_i^{alt} - 2.12$ (mGal).

Option 2: $\delta_i^{alt} = \widehat{\delta}_i^{alt} - 2.27$ (mGal).

Option 3: $\delta_i^{alt} = \widehat{\delta}_i^{alt} - 2.16$ (mGal).

4.2.3.2. Fitting second time use to the Least Squares Collocation method.

a) Determination of analytic covariance function parameters from residual gravity anomalies.

The results of determining the parameters of the analytic covariance function of options are given in Table 4.9

Table 4.9. The results of determining the parameters of the analytic covariance function of options

Options	***N***	***a***	***R_B-R* (*km*)**	**A (m/s)^4**	**Gravity anomaly variance (mGal^2)**
Option 1	315	33.9946	-0.99997	$0.1217.10^{-2}$	74.77
Option 2	312	34.6422	-0.99993	$0.6306.10^{-2}$	72.32
Option 3	315	32.9702	-0.99997	$0.1217.10^{-2}$	72.52

b) Determination of the residual gravity anomalies after fitted with the residual ship-measured gravity anomalies.

The results of the residual marine gravity anomalies after fitted with the residual ship-measured gravity anomalies on the Gulf of Tonkin - Vietnam according to the options shown in Table 4.10.

Table 4.10. Statistics summarizing the results of the residual marine gravity anomalies after fitted with the residual ship-measured gravity anomalies

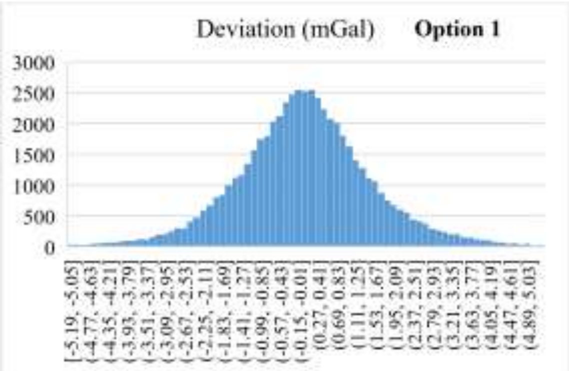
Options	**Satellites**	**Grid**	**Tide System**	**Residual marine gravity anomalies after fitted**		
				Mean (mGal)	**Min (mGal)**	**Max (mGal)**
Option 1	Cryosat-2/GM	2′x2′	Tide free	-2.91	-22.14	+21.33
Option 2	Saral/AltiKa	2′x2′		-3.21	-23.13	+22.91
Option 3	Cryosat-2/GM & Saral/AltiKa	1.5′x1.5′		-2.98	-23.84	+23.61

c) Assess the accuracy of the residual gravity anomalies after fitted with the residual ship-measured gravity anomalies.

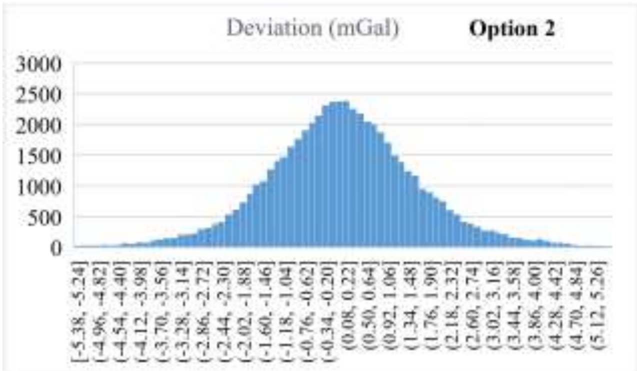
The accuracy assessment results are shown in Table 4.11. The frequency chart of deviations between the residual marine gravity anomalies after fitted with the residual ship-measured gravity anomalies are shown in Figure 4.4; Figure 4.5; Figure 4.6 shows: deviation follows the rule of random error.

Table 4.11. The results of the assessment of the accuracy of the calculation options after fitted the ship-measured gravity anomalies on the Gulf of Tonkin - Vietnam.

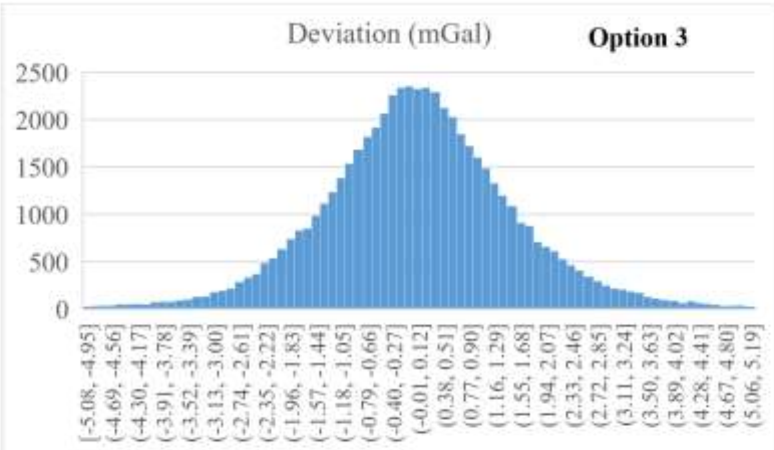
No.	Statistics	Option 1	Option 2	Option 3
1	Min. Dev: $\Delta_{\delta g}^{min}$ (mGal)	-5.19	-5.38	-5.08
2	Max. Dev: $\Delta_{\delta g}^{max}$ (mGal)	+5.29	+5.50	+5.18
3	Mean. Dev: $\Delta_{\delta g}^{tb}$ (mGal)	+0.05	+0.07	+0.06
4	Std. Dev: $\sigma_{\delta g}$ (mGal)	±1.48	±1.54	±1.45



Histogram 4.4. The deviation histogram for differences between satellite-derived gravity anomalies and ship-measured gravity anomalies and after fitted (Option 1).



Histogram 4.5. The deviation histogram for differences between satellite-derived gravity anomalies and ship-measured gravity anomalies and after fitted (Option 2).



Histogram 4.5. The deviation histogram for differences between satellite-derived gravity anomalies and ship-measured gravity anomalies and after fitted (Option 3).

The comparative results showed that: the marine gravity anomalies after fitted with the ship-measured gravity anomalies has almost no system error and the accuracy is achieved: ±1.48 mGal (Option 1); ±1.54 mGal (Option 2); ±1.45 mGal (Option 3).

Thus, the results of option 3 have the highest accuracy. The residual marine gravity anomalies after fitted is more accurate than before fitting: ±1.45mGal (Option 3). The system error between satellite-derived gravity anomalies and the ship- measured gravity anomalies has also decreased from +2.16 mGal to approximately 0.0 mGal (+0.06) (Option 3). This proved that the solutions proposed by the thesis have improved the accuracy of determining satellite-derived gravity anomalies.

After to have the results of the residual marine gravity anomalies after fitted (δg) (option 3 – the most accurate option) , proceed to restore the long-wavelength gravity anomaly Δg_{EGM} from the fully normalized gravitational coefficients of global geopotential model (EIGEN-6C4) and determine gravity anomalies after fitted by formula (2.2.8).

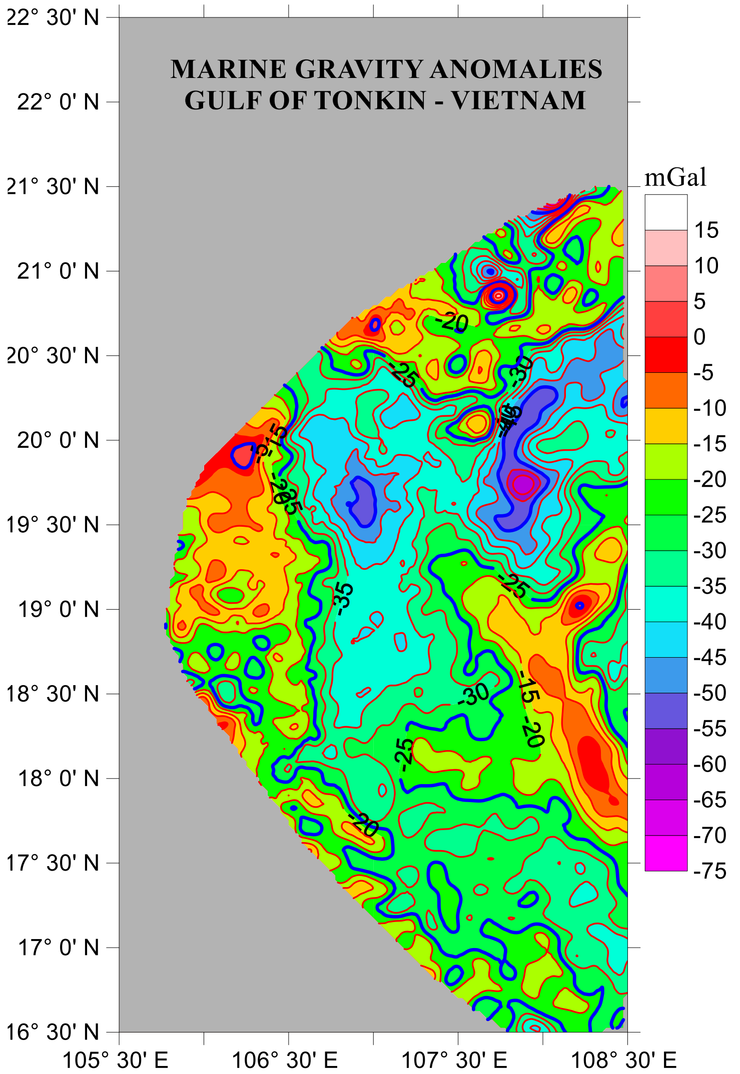


Figure 4.10. The results of satellite-derived gravity anomalies of Cryosat - 2/GM and Saral/Altika (Option 3) after fitted with the ship-measured gravity anomalies represented by the level and colors on the Gulf of Tonkin - Vietnam.

4.3. Assess the accuracy of the results of satellite-derived gravity anomalies available on the Gulf of Tonkin - Vietnam.

In order to compare the accuracy of the marine gravity anomalies determined by the Ph.D. student with the results of previous research on the Gulf of Tonkin - Vietnam. Using 56978 points ship-measured gravity anomalies to evaluate the accuracy of global gravity anomaly models determined by satellites altimetry data (DTU10GRA, DTU13GRA, DTU15GRA, DTU17GRA) on the Gulf of Tonkin - Vietnam. Assessment results are shown in Table 4.14.

Table 4.14. Statistics of the results of the evaluation of the accuracy of the global gravity field models DTU10GRA, DTU13GRA, DTU15GRA, DTU17GRA and the results of the thesis on the Gulf of Tonkin - Vietnam

No.	Models	Mean. Dev $\Delta_{\delta g}^{tb}$ (mGal)	Std. Dev $\sigma_{\delta g}$ (mGal)
1	DTU10GRA	+2.99	±5.80
2	DTU13GRA	+2.95	±5.73
3	DTU15GRA	+3.19	±5.63
4	DTU17GRA	+3.11	±5.76
5	Phương án 1	+0.05	±1.48
6	Phương án 2	+0.07	±1.54
7	Phương án 3	+0.06	±1.45

Comparing the results of the 3 options of the thesis with the results calculated by the world shown in Table 4.14 on the Gulf of Tonkin - Vietnam shows that: After applying the solutions to improve accuracy, the results of satellite-derived gravity anomalies are better than, the error is evaluated according to the standard deviation: ±1.45mGal. The works of the world, the best results achieved have an error of evaluation according to the standard deviation: ±5.63mGal (DTU15GRA). This confirmed that the solutions proposed by the thesis are theoretically rigorous and completely feasible in empirical calculations, ensuring improved accuracy in determining marine gravity anomalies by satellite altimetry.

4.4. Conclusion of Chapter 4

- Using 105 cycles of the Cryosat-2/GM satellite; Global Geopotential Model (EIGEN-6C4) and Mean Dynamic Topography model (DTU15MDT) to determine marine gravity anomalies by the Least Squares Collocation method. The achieved marine gravity anomaly has an accuracy assessed by standard deviation: ±2.80 mGal. After fitted with the ship-measured gravity anomalies, the system error for the entire study area has been removed, the accuracy achieved for the area within and away from the ship-measured gravity anomalies 0.5^0 is ±1.48 mGal;
- Using 54 cycles of the Saral/AltiKa satellite; Global Geopotential Model (EIGEN-6C4) and Mean Dynamic Topography model (DTU15MDT) to determine marine gravity anomalies by the Least Squares Collocation method. The achieved marine gravity anomaly has an accuracy assessed by standard deviation: ±2.89 mGal. After fitted with the ship-measured gravity anomalies, the system error for the entire study area has been removed, the accuracy achieved for the area within and away from the ship-measured gravity anomalies 0.5^0 is ±1.54 mGal;
- Combine 105 cycles of the Cryosat-2/GM satellite with the 54 cycles of the Saral/AltiKa satellite; Using Global Geopotential Model (EIGEN-6C4) and Mean Dynamic Topography model (DTU15MDT) to determine marine gravity anomalies by the Least Squares Collocation method. The achieved marine gravity anomaly has an accuracy assessed by standard deviation: ±2.77 mGal. After fitted with the ship-measured gravity anomalies, the system error for the entire study area has been removed, the accuracy achieved for the area within and away from the ship-measured gravity anomalies 0.5^0 is ±1.45 mGal;

The results of the satellite-derived gravity anomalies by option 3 (combining the data of the Cryosat-2/GM satellite with the data of the Saral/AltiKa satellite) were more accurate than the other two options. The solutions proposed by the thesis have a coherent theory, which has improved the accuracy of determining marine gravity anomalies by satellite altimetry.

CONCLUSIONS AND RECOMMENDATIONS

1. Conclusions

The content of the thesis has achieved the set goal of researching solutions to improve the accuracy of gravity anomalies are identified by satellite altimetry data. The results achieved and new contributions to the science of the thesis can be mentioned as follows:

a) The thesis proposed solutions to improve the accuracy of gravity anomalies identified by satellite altimetry data, namely:

- Select data of satellites altimetry performing geodetic missions and combination to receive high-precision Sea Surface Height (SSH) data as well as increase the density of points for the research area;
- Using GPS/leveling data to evaluate and select the Global Geopotential Model (GGM) most suitable for the territory of Vietnam (EIGEN6-C4) to receive the high-precision long-wavelength geoid height (NEGM) for use in the "remove - restore" technique;
- Using the tidal gauge data along the coast of Vietnam to evaluate and select the Mean Dynamic Topography models (MDT) most suitable Vietnam (DTU15MDT) for use in calculations;
- Analysis, comparison of methods, and selection of the Least Squares Collocation method as the best method for converting the residual geoid heights into residual gravity anomalies;
- Fitting the satellite-derived gravity anomalies with ship-measured gravity anomalies in 2 steps: (1) correct system deviations; 2) fitting random errors using the Least Squares Collocation method.
- These solutions are theoretically logical, and viable and have improved the accuracy of satellite-derived gravity anomalies.

b) The thesis has developed the GEOMAT2015 computer program that allows the identification of geoid height, height anomalies, and gravity anomalies from the fully normalized gravitational coefficients of global for experimental calculation of solutions to improve the accuracy of marine gravity anomalies by satellite altimetry data. The results calculated using this program are highly accurate and reliable.

c) The thesis has identified marine gravity anomalies on the sea Gulf of Tonkin -Vietnam from satellite altimetry data cryosat-2/GM combined with Saral/Altika in geodetic missions by the Least-Squares Collocation method, using the EIGEN6-C4 model, Mean Dynamic Topography model DTU15MDT and fitting with ship-measured gravity anomalies. The identified marine gravity anomaly has a density (1.5' x 1.5') and is assessed according to the standard deviation compared with the ship-measured gravity anomalies of ±2.77 mGal. After fitted with the ship-measured gravity anomalies, the system error for the entire study area has been removed, the accuracy achieved for the area within and away from the ship-measured gravity anomalies 0.5^0 is ±1.45 mGal. The results of this thesis are better than other studies in the coastal area.

2. Recommendations for further research

- From the research results of the thesis, further research should be carried out on other seas, especially nearshore waters and shallow waters that are subject to many complex waves, wind, tidal and geophysical factors
- It is possible to apply the solutions proposed by the thesis to build a set of marine gravity anomalies for the whole East sea using satellite altimetry data.

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